13 Event reconstruction

13.1 RICH ring finding

13.1.1 Elastic Net for standalone RICH ring finding

The elastic net method [24] is a kind of artificial neural network [25, 26] that has been used for track recognition in high energy physics [2, 4, 27, 28]. The elastic net algorithm is one of the best optimization algorithms in terms of efficiency and speed.

The method is well illustrated on a simple example of the traveling salesman problem (TSP). The traveling salesman problem is a classic problem in the field of combinatorial optimization, in which efficient methods for maximizing or minimizing a function of many independent variables is sought. The problem is to find for a number of cities with given positions the shortest tour in which each city is visited once.

All exact methods known for determining an optimal route require a computing effort that increases exponentially with the number of cities, so in practice exact solutions can be attempted only on problems involving a few hundred cities or less. The traveling salesman problem thus belongs to the large class of nondeterministic polynomial time complete problems. Many heuristic algorithms were developed for the TSP aiming to bypass the combinatorial difficulties [29]. One of the most successful approaches to the problem is the elastic net of Durbin and Willshaw [24]. The elastic net can be thought of as a number of beads connected by elastics to form a ring. The essence of the method is to iteratively elongate a circular close path in a non uniform way until it eventually passes sufficiently near to all the cities to define a tour.

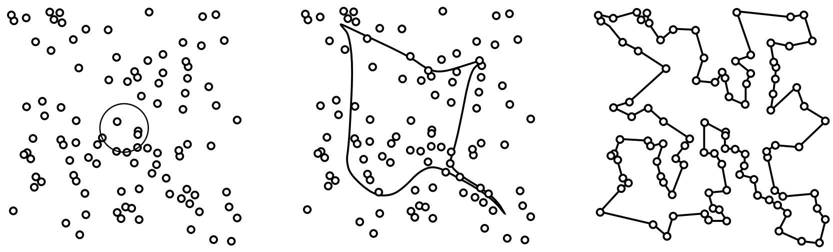

Figure 13.1: Example of the progress of the elastic net method in the traveling salesman problem with 100 cities [24]

Following the deformable template approach [25], let us denote the cities by $\vec{x}_i$. We are going to match these cities with template coordinates $\vec{y}_a$ such that $\sum_a |\vec{y}_a - \vec{y}_{a+1}|$ is minimum and that each $\vec{x}_i$ is matched by at least one $\vec{y}_a$. Define a binary neuron s_{ia} to be 1 if a is matched to i and 0 otherwise. The following energy expression is to be minimized in a valid tour:

$$E\left(s_{ia}, \vec{y}_a\right) = \sum_{ia} s_{ia} \cdot |\vec{x}_i - \vec{y}_a|^2 + \gamma \cdot \sum_{a} |\vec{y}_a - \vec{y}_{a+1}|^2 . \tag{13.1}$$

The multiplier γ governs the relative strength between matching and tour length. Applying the mean field approximation [25] one can derive the dynamical equation:

$$\Delta \vec{y}_a = \eta \left[2 \sum_i v_{ia} \cdot (\vec{x}_i - \vec{y}_a) + \gamma \cdot (\vec{y}_{a+1} - 2\vec{y}_a + \vec{y}_{a-1}) \right] , \tag{13.2}$$

where continuous neurons v_{ia} describe matching of a to i:

$$v_{ia} = \frac{e^{-|\vec{x}_i - \vec{y}_a|^2/T}}{\sum_b e^{-|\vec{x}_i - \vec{y}_b|^2/T}}\,. \tag{13.3}$$

Here the "temperature" T is decreasing at each update of templates $\vec{y}_a$, and η is the parameter controlling the minimization speed.

The algorithm is thus a procedure for the successive recalculation of the positions of a number of points of the plane in which the cities lie. The points describe a closed path which is initially a small circle centered on the middle of the distribution of cities and is gradually elongated non-uniformly to eventually pass near all the cities and thus define a tour around them, see Fig. 13.1 (for details see the original paper [24]). Each point on the path moves under the influence of two types of force (see Eq. 13.2):

1. the first moves it towards those cities to which it is nearest;

2. the second pulls it towards its neighbors on the path, acting to minimize the total path length.

By this process, each city becomes associated with a particular section on the path. The tightness of the association is determined by how the force contributed from a city depends on its distance, and the nature of this dependence changes as the algorithm progresses. Initially all cities have roughly equal influence on each point of the path. Subsequently a larger distance becomes less favored and each city gradually becomes more influenced by the points on the path closest to it.

The elastic net algorithm produces tours of the same quality as other well known heuristic algorithms [30].

Evolution of the elastic net coordinates in continuous space results in significantly large number of iterations without changing the order of the cities. This can be avoided if the net nodes will force to coincide with cities at each iteration. Such modification of the elastic net has been developed by us in order to increase speed of the net. In this so-called discrete algorithm the elastic net can be represented as a closed tour passed exactly through a subset of the cities. An iteration consists of adding new cities to or releasing some cities from the net.

File name	Number of cities	Extra path (%)	Time, ms	Time per city, μs
berlin52	52	0.00	0.98	19
st70	70	4.27	1.27	18
kroA100	100	3.03	1.46	15
lin105	105	0.78	1.84	18
ch130	130	5.59	2.56	20
tsp225	225	5.34	4.36	19
pcb442	442	8.37	12.35	28
pr1002	1002	6.12	24.94	25
pr2392	2392	8.42	58.53	24

Table 13.1: Extra path length (in % to the optimum) and time (in ms) of the discrete ENN algorithm in the TSP problem for several distributions of cities with known optimal tour

Results of application of the discrete ENN algorithm to several distributions of cities with known optimal tour are presented in Table 13.1. The algorithm has good performance for real-life applications and is extremely fast. Numbers in the last column of the table show almost constant execution time per city that means linear behavior of the algorithm with respect to increase of the combinatorial complexity of the problem.

Based on the discrete elastic net we have developed an algorithm for standalone RICH ring reconstruction. The algorithm has been already successfully tested on RICH data of the COMPASS experiment [31]. Here we focus on maximizing the speed of the algorithm aiming its implementation in the Level-1 trigger. The task of ring finding in the RICH detector with about 1000 hits per event is similar in combinatorial complexity to the `pr1002` example in the table 13.1. Having now additional knowledge of the form of the final tour one can expect increase of the speed down to a few milliseconds per event.

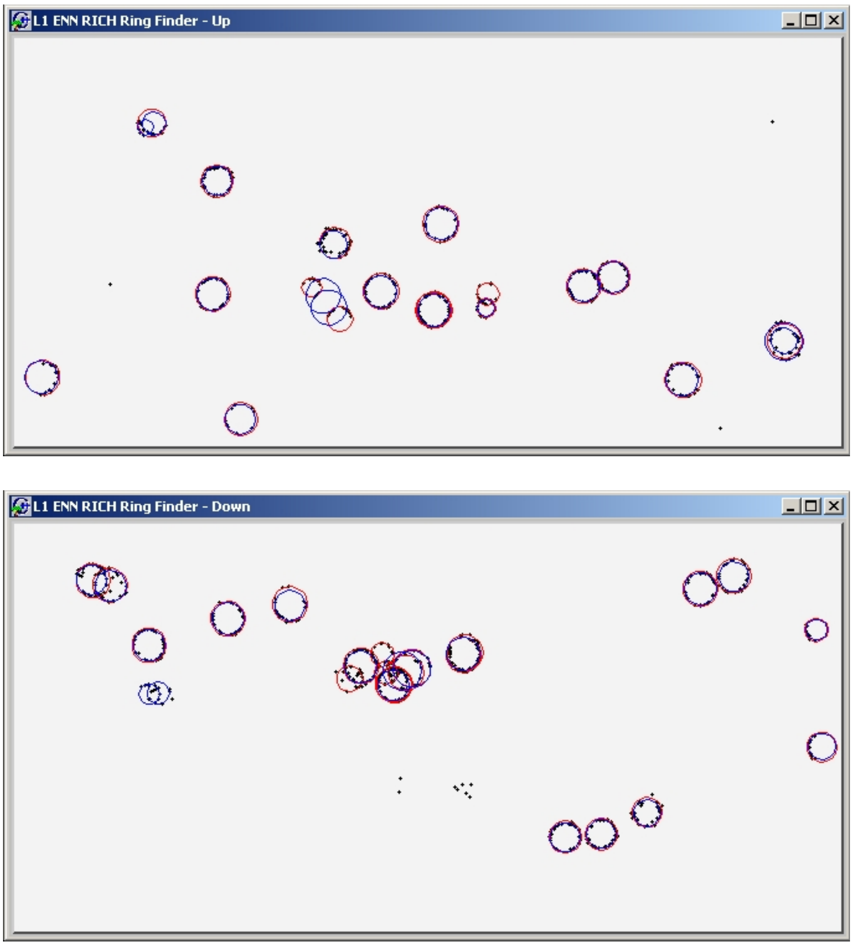

Figure 13.2: Example of a RICH event reconstructed by the L1ENNRingFinder algorithm. Upper and bottom parts of the RICH detector are shown separately. Reconstructed rings are blue, while Monte Carlo rings are drawn in red color (with slightly scaled radius to avoid overlapping with the reconstructed rings).

The task is to reconstruct rings by measured hits on a detection plane of the RICH detector. As circular form of rings is predefined there is no need in internal forces of the elastic net. In contrast to the TSP problem here the net does not pass through all hits. The task is to find rings surrounded by maximum hits within errors of measurements. The interaction force between hits and the net does not depend on distance. In this case noise hits do not attract the net making the algorithm robust. The net converges to the area of maximum condensation of hits within 2–3 iterations, therefore the total time to reconstruct an event is proportional to:

$$T_{\text{L1ENN}} \sim N_{\text{rings}} \cdot N_{\text{hits per ring}}. \quad (13.4)$$

Since the rings are independent, the algorithm reconstructs them one by one[1]. The elastic net algorithm performs searching for a ring in a local area of the detector plane. When the ring found by the elastic net

[1] In a hardware implementation the algorithm can run in parallel several elastic nets.

is accepted, all hits belonging to the ring are marked as used. The algorithm repeats until it recognizes rings among hits left on the plane. Rings are fitted at the step of searching. Example of a reconstructed event is given in Fig. 13.2.

Rings set	Performance (%)	Number of rings
Reference set efficiency	92.21	1425
All set efficiency	80.52	4179
Extra set efficiency	74.47	2754
Clone rate	3.26	142
Ghost rate	14.98	652
Found MC rings/event	33	
Time/event (ms)	1.07	

Table 13.2: Performance of the L1ENNRingFinder algorithm taken on 100 events of central Au-Au collisions at 35 AGeV

The performance of the L1ENNRingFinder algorithm is presented in Table 13.2. The "all set" contains rings with more than 5 hits. In the "reference set" we put rings which originate from the target region and have more than 15 hits. The other rings form the "extra set".

A reconstructed ring is assigned to a generated Monte Carlo ring if there is at least 70% hits correspondence. A Monte Carlo ring is regarded as found if it has been assigned to at least one reconstructed ring. If the ring is found more than once, all additionally reconstructed rings are regarded as clones. A reconstructed ring is called ghost if it is not assigned to any Monte Carlo ring using 70% criteria.

The L1ENNRingFinder algorithm shows a very good efficiency for reference rings (92%). Ghost rate will be suppressed at the next step when rings will be matched to tracks from other detectors.

Because of its computational simplicity and extremely high speed (1 ms), the algorithm is considered to be further implemented in hardware which can increase the speed by another few orders of magnitude.

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