Study of different expressions for Δ Dalitz decay dilepton mass spectra

In this report, we test the sensitivity of the e^+e^- invariant mass spectrum to three different formula of the Δ Dalitz decay ($\Delta^+ \to pe^+e^-$) width used in recent calculations of dilepton spectra in C+C at 1 and 2 A.GeV.

- The first one ("Ernst" formula) comes from [1] and is used in the most recent paper of E. Bratkovskaia et al [2]. It is also used in our PLUTO generator.
- The second one ("Wolf" formula) comes from [3] and is used in the recent paper of the Nantes group [4]. It was also used in the papers of Bratkovskaya et al. until 2007.
- The third one ("Krivoruchenko" formula) was given in [5] and is used by the Tübingen group ([6],[7]) .

In Sec. 1 and Sec. 2, we compare the dilepton spectra produced using the different formula. In Sec. 3, we use the radiative Δ decay width values to test them. In Sec. 4, we discuss the different form factors used and in Sec.5, we deduce the different branching ratios for Δ Dalitz decay and conclude about Pluto inputs.

1 Difference between the three formulas

The difference between these expressions was stressed in Krivoruchenko et al [5]. Ernst and Krivoruchenko calculate the differential decay width as a function of the dilepton (or γ^*) mass m from the $\Gamma_{\Delta\to N\gamma^*}$ width in the following way (factorization hypothesis):

$$\frac{d\Gamma_{\Delta\to Ne^+e^-}}{dm} = \frac{2\alpha}{3\pi m}\sqrt{1-\frac{4m_e^2}{m^2}}\left(1+\frac{2m_e^2}{m^2}\right)\Gamma_{\Delta\to N\gamma^*}, \tag{1}$$

Wolf uses a simpler expression:

$$\frac{d\Gamma_{\Delta\to Ne^+e^-}}{dm} = \frac{2\alpha}{3\pi m}\Gamma_{\Delta\to N\gamma^*} \tag{2}$$

which however makes no difference, as soon as the dilepton mass is well above the threshold (m>> $2m_e$). The difference between the three formulas therefore comes from the expression of $\Gamma_{\Delta\to N\gamma^*}$.

There are three independent helicity amplitudes for the $\Delta \to N\gamma^*$ transition. Krivoruchenko et al. take into account the three of them, and the decay width therefore depends on the three transition form factors $|G_M(m)|$ (magnetic), $|G_E(m)|$ (electric) and $|G_C(m)|$ (Coulomb) , which are calculated in the extended VDM model, whereas Ernst and Wolf take only the magnetic coupling with a constant G_M value.

2 Comparison of the Δ Dalitz decay dilepton mass spectra

In order to test the sensitivity to the different Δ Dalitz decay width formula,s independantly of the assumptions made by the different authors on transition form factors, we used in each case a constant G_M value equal to 2.7 and G_E and G_C equal to zero.

The corresponding dilepton mass spectra are shown in fig.1 and the ratios of the yield obtained with the Ernst formula to the ones obtained respectively with the Wolf and Krivoruchenko formula in fig.2. These ratios are respectively about 1.6 and 4.75 and show only a very small $M_{e^+e^-}$ dependence. These formulas therefore differ mainly by a normalisation factor in the $\Gamma_{\Delta\to N\gamma^*}$ expression, which can be derived from the value at the photon point $M_{\gamma^*} = 0$.

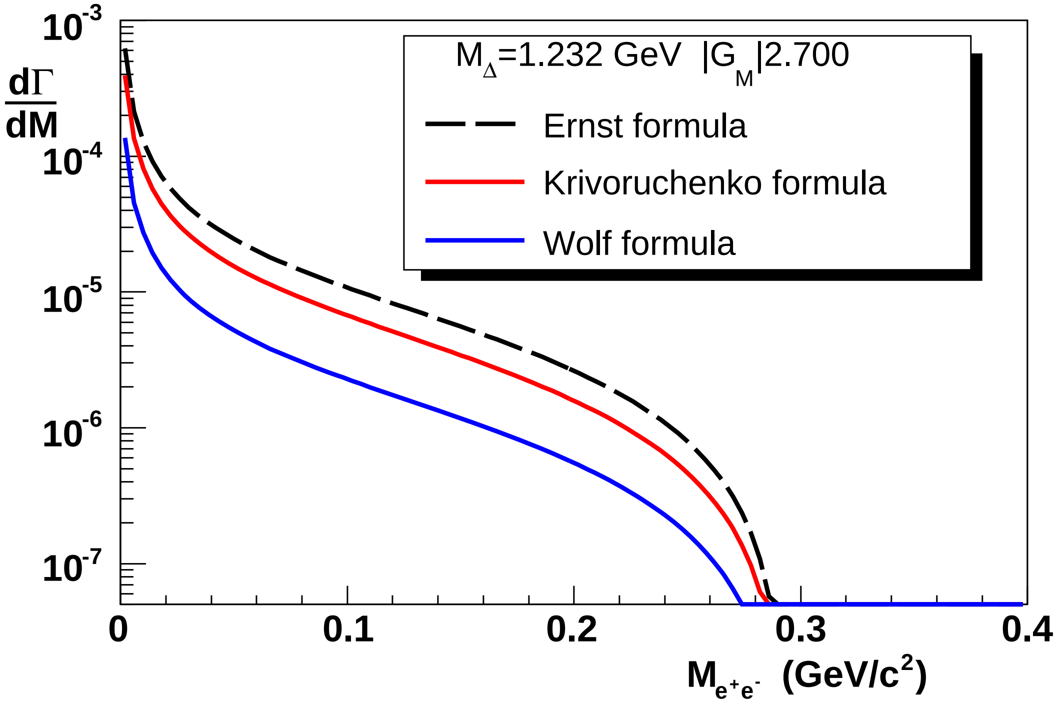

Figure 1: *differential Delta Dalitz decay width as a function of dilepton invariant mass using the three different formulas. In each case, the same constant magnetic form factor* $|G_M|$*=2.7 is taken.*

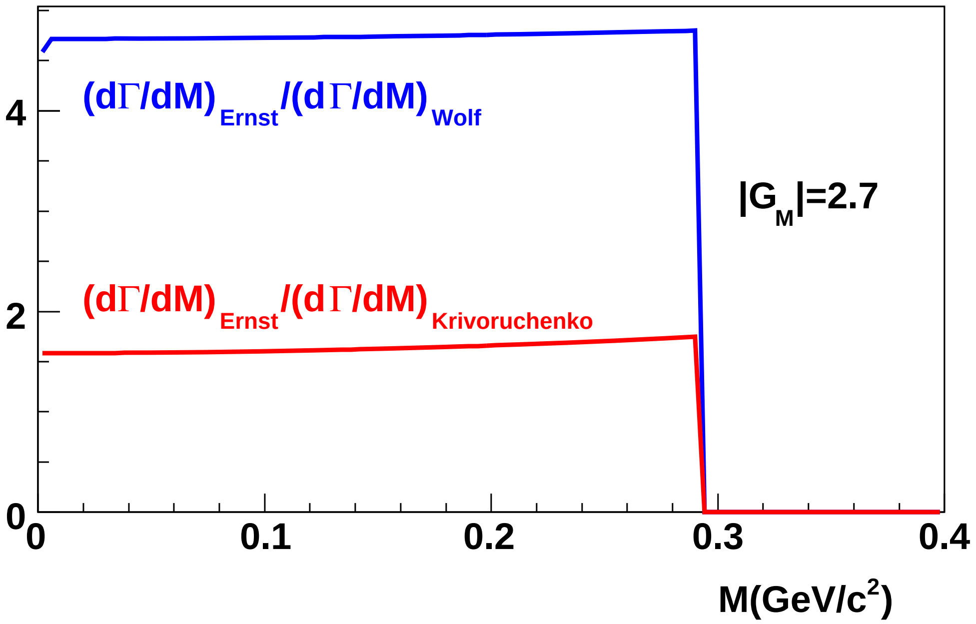

Figure 2: *Ratios between Ernst and Wolf formula (blue) and Ernst and Krivoruchenko formula(red) as a function of the dilepton mass. In each case a constant magnetic form factor* $|G_M|$*=2.7 is used.*

3 Δ radiative decay width

At M_{γ^*}=0 (photon point), the $\Delta^+ \to p\gamma^*$ decay width coincides with the radiative decay width $\Delta^+ \to p\gamma$, which is known experimentally from pion photoproduction experiments:

$$\Gamma_{\Delta^+ \to p\gamma} = 0.6 - 0.72 MeV, BR = 0.52 - 0.60\% (PDG2006) \tag{3}$$

The most precise measurement of $G_M(0)$ gives $G_M(0)$= 3.00 ± 0.05 [8]. The ratio of electric to magnetic form factors is also known quite precisely: $G_E(0)/G_M(0)$=-2.5%, which makes a contribution to the radiative decay width lower than 2 10^{-3}. The radiative decay width is therefore mainly a function of $G_M(0)$.

Turning back to our papers on Δ Dalitz decay, the relation between $\Gamma_{\Delta \to N\gamma}$ and $G_M(0)$ can be deduced in each case, assuming $G_E(0)$=$G_C(0)$=0:

$$\Gamma^{Ernst}_{\Delta \to N\gamma} = \frac{\alpha \left|G_M(0)\right|^2}{16} \frac{(m_\Delta - m_N)^3 \left(7m_\Delta^4 + 8m_\Delta^3 m_N + 2m_\Delta^2 m_N^2 + 3m_N^4\right)}{m_\Delta^3 m_N^2 \left(m_\Delta + m_N\right)} \tag{4}$$

”‘Ernst” (4)	”Wolf” (5)	”Krivoruchenko” (6)	experiment
1.03 MeV	0.22 MeV	0.65 MeV	0.66 ± 0.06 MeV

Table 1: *Radiative Δ decay widths deduced from the expressions (4),(5),(6) using the experimental value $G_M(0)=3.00\pm0.05$.*

Numerically, $\Gamma^{Ernst}_{\Delta\to N\gamma} = 0.114\,|G_M(0)|^2$ MeV at Δ pole mass.

$$\Gamma^{Wolf}_{\Delta\to N\gamma} = \frac{\alpha\,|G_M(0)|^2}{16}\frac{(m_\Delta - m_N)^3\left(7m_\Delta^4 + 8m_\Delta^3 m_N - 2m_\Delta^2 m_N^2 + 3m_N^4\right)}{4m_\Delta^3 m_N^2\,(m_\Delta + m_N)} \tag{5}$$

Numerically, $\Gamma^{Wolf}_{\Delta\to N\gamma} = 0.024\,|G_M(0)|^2$ MeV,
One can notice that the expression of $\Delta^+ \to \gamma$p given by Ernst differs from Wolf's by a factor 4 and by a sign difference in one factor of the numerator.

On the other hand, using the formula by Krivoruchenko et al. [5], one obtains for the radiative decay:

$$\Gamma^{Krivoruchenko}_{\Delta\to N\gamma} = \frac{\alpha\,|G_M(0)|^2}{16}\frac{(m_\Delta - m_N)^3\,(m_\Delta + m_N)^3}{m_\Delta^3 m_N^2} \tag{6}$$

Numerically, $\Gamma^{Krivoruchenko}_{\Delta\to N\gamma} = 0.072\,|G_M(0)|^2$ MeV

The ratios between these numbers are consistent with fig. 2. In table 1, the radiative decay widths are calculated from the three formula using the experimental value $G_M(0)=3.00$ and compared to the experimental radiative decay width.

The conclusion is that only Krivoruchenko's formula reproduces the experimental value with a good precision. This is a test that formula (6) is coherent with the definition of G_M used in the analysis of pion photoproduction or electroproduction data. It is therefore expected that the normalisation of the dilepton spectrum should be correct using the Krivoruchenko's formula, if a $G_M(0)$ value close to 3 is used.

However, the different authors also use various G_M values, which changes the normalisation of the dilepton spectrum. We investigate this in the next section.

4 G_M values used in the calculations

We have seen that the different formulas of the differential Dalitz decay width differ only by a normalisation factor. The latter is in principle fixed by the radiative decay width. We use therefore the values of radiative decay width as a reference to compare the normalisation provided by the combination of different formula and choice of G_M values. In each case, we also check that we can reproduce the dilepton spectra shown in the different papers, with the formula and values of form factors given by the authors.

We could reproduce the spectrum in fig.2 of [1] using "Ernst" formula and $|G_M(m)|$=$|G_M(0)|$=3.0, as indicated in the paper. This value is in agreement with the latest Δ photoproduction experiments and is also used in the most recent Bratokvskaya's paper [2]. But, as was shown in Sec. 3, the normalisation of the Ernst's formula is either wrong or not consistent with this definition of form factors and it yields $\Gamma_{\Delta^+\to p\gamma}$=1.03 MeV, which is too high by a factor 1.6. So, the whole dilepton mass spectrum is overestimated by the same factor.

The value $|G_M|$=2.7 is used by Wolf[3]. However, injecting this value in Wolf's expression yields a radiative decay of about 0.18 MeV (i.e a branching ratio of 1.5 10^{-3}), which is much too low (see table 2). However, this normalisation problem has probably been noticed, and the dilepton spectrum shown in fig. 6 of [3] corresponds in fact to the calculation with $|G_M|$=5.44 ($\sim$2*2.7), which is the value mentionned in Bratkovskaya's papers. This means that this value was taken already in [3], or equivalently that the normalisation of the "Wolf's" formula was changed by a factor 4. With formula(5), we obtain a radiative decay width of 0.71 MeV. Such a value was in the 90's the PDG value and is now the upper limit for radiative decay width, so the normalisation of the dilepton spectrum is consistent.

In their very recent paper, Thomere et al. [4] write again the same "Wolf's" Dalitz decay formula and use $|G_M|$=2.7. If this is really the case, they underestimate the radiative Δ decay and therefore the Δ Dalitz decay by about a factor 3.7. It is however likely that they have corrected the normalisation as was done in the older paper [3].

We have seen in Sec. 3 that the "Krivoruchenko" formula gives the correct relation between radiative decay width and magnetic form factor at the photon point. The Tübingen group uses this formula with electromagnetic transition form factors from the extended VDM model, which are fitted to some space-like transition form factors data existing by year 2000. We estimated from (6) the value $|G_M(0)|$=3.02 reproducing the experimental radiative decay width (table 2). Assuming that the effect of the dilepton mass dependence of the form factor is small, we took a constant magnetic form factor equal to 3.02 and injected it in Krivoruchenko's formula. In this way, we could reproduce the dilepton spectrum in fig. 20 of [9].

However, in this paper and in later papers of the Tübingen group (for example [7]), the whole expression of Δ Dalitz decay dilepton mass distribution is multiplied by 3, with respect to [5], see for example eq. (III.22) of [9] or eq. (5) of [7]. This factor 3 is probably a misprint. Our checks indeed show that the dilepton spectrum from Δ(1232) Dalitz decay of fig. 26 of [9] doesn't take this factor 3 into account.

So, we could reproduce the normalisation of the different dilepton spectra shown by the different authors, Ernst spectrum is too high by a factor 1.6. Wolf's formula gives a correct yield, but seems to use a renormalisation of the magnetic form factor. Only the "Krivoruchenko's" calculation has both the correct normalisation and a consistent photon-point magnetic transition form factor value.

5 Dalitz decay branching ratio

In our Pluto event generator, the formula giving the Δ Dalitz decay ($\Delta^+ \to pe^+e^-$) branching ratio is taken from Ernst, but with $|G_M|$=2.7. This normalisation factor has however no influence, since

	Wolf[3],[4] litt. form.	Wolf [3],fig.6 Bratk. <2007 (e.g. [10])	Ernst [1] Bratk. >2007 [2]	Kriv.[5] const. G$_M$	Kriv.[5] e-VDM	PLUTO	exp.
G$_M$	2.7	5.44	3.0	3.02	e-VDM		3.00±0.05
$10^3 BR_{\Delta \to N\gamma}$	1.5	6.0	8.7	5.6			5.6±0.4
$10^5 BR_{\Delta Dalitz}$	1.15	4.6	6.5	4.12	4.25	4.4	?

Table 2: *G_M values used in the different papers. Corresponding radiative and Delta Dalitz branching ratios. The numbers quoted in the first column refer to the litteral formula written in the papers, but the normalisation of fig. 6 of [3] corresponds to the second column. The fifth column corresponds to the Krivoruchenko's formula in [5] (and not to the ones in [9] and [7] which has a factor 3 difference) and to a constant G_M=3.02. which reproduces both the radiative Δ decay width and the normalisation of the spectrum in fig.26 of [9]. The Δ Dalitz decay width of table VII of [9] calculated with the e-VDM form factors is given in column 6. The G_M PLUTO value and the radiative decays PLUTO values are not quoted, since they are not correlated to the Dalitz decay branching ratio.*

the yields are normalized to the Dalitz decay branching ratio given as a separate parameter and set to 4.4 10^5.

This branching ratio can be deduced by integrating the different expressions of $d\Gamma_{\Delta \to Ne^+e^-}/dm$. Taking Krivoruchenko's formula and $|G_M|$=3.02, which, as shown before, gives the correct value for the Δ radiative decay, we obtain, for a Δ mass=1232 MeV/c^2, Γ= 4.9 keV , i.e. BR=4.12 10^{-5}, which is comparable to the value 5.02 keV, i.e. BR=4.25 10^{-5}, quoted in table VII of [9] and obtained with the extended VDM form factors. This shows that these form factors have a very small effect on the branching ratio. The value used in the PLUTO generator is only about 7% higher than the value calculated with Krivoruchenko's formula and a constant G$_M$ value=3.02 consistant with photoabsorption experiments.

The other formulas give various values for the branching ratio which are listed in the last line of table 2. Except the litteral "Wolf's" formula which probably doesn't correspond to the real normalisation of the calculations, the highest discrepancy is for Erns't formula which provides a normalisation of the dilepton spectrum which is a factor 1.6 higher.

6 Δ mass dependence on dilepton yield

The different expressions of Δ Dalitz decay width are dependent on the Δ mass. This is illustrated in fig. 3. In this picture, the three different formulas are used here with G$_M$ factors consistent with the Δ Dalitz decay dilepton yield shown in the different papers. i.e. "Ernst" formula is used with G$_M$=3 (fig.2 of [1]), "Wolf " formula is used with G$_M$=5.44 (fig.6 of [3]), "Krivoruchenko" formula with G$_M$=3.02 (fig. 26 of [9]), so they should correspond to the inputs of the transport models.

The general feature is an overall increase of the decay width and a broadening of the differential Dalitz decay width towards large dilepton masses as expected from the phase space. This induces a sensitivity of the Δ Dalitz decay dilepton yield on the Δ mass distribution. For information, the highest mass available in pp reaction at 1.25 GeV is about 1.48 GeV/c^2 and about 1.82 GeV/c^2 at 2.2 GeV.

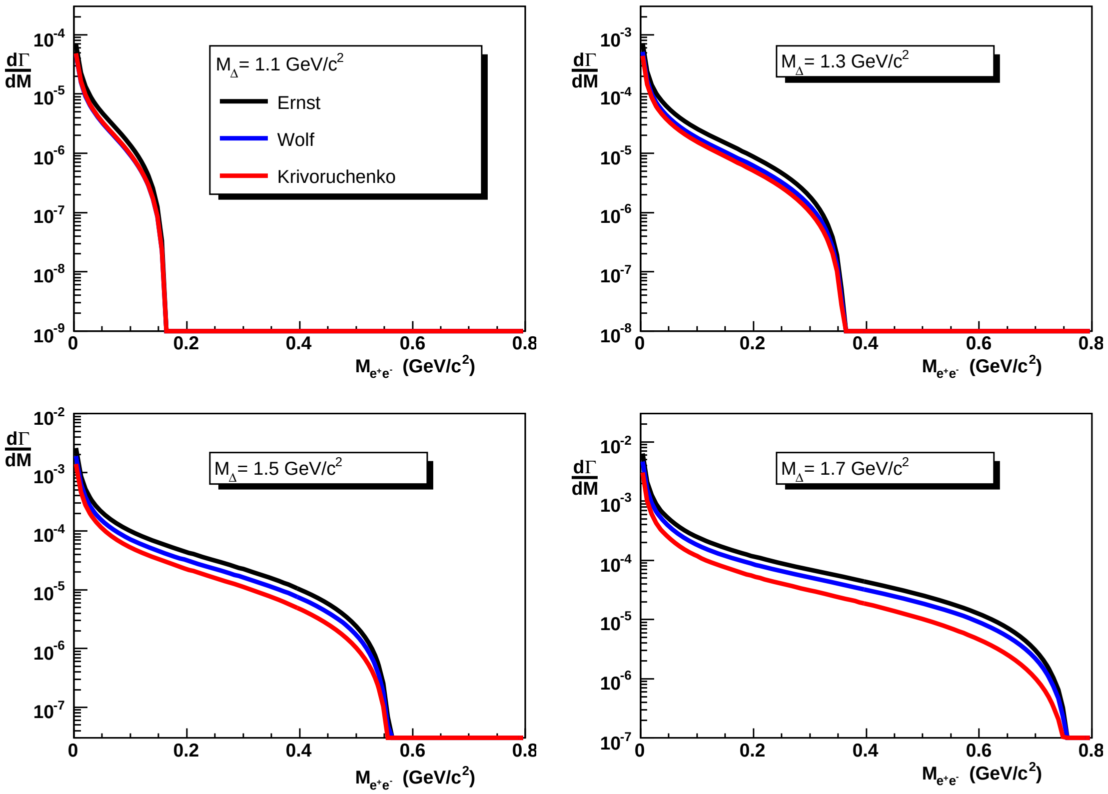

Figure 3: *Dalitz decay width as a function of dilepton invariant mass for different* Δ *masses. The three different formulas are used here with* G_M *factors consistent with the* Δ *Dalitz decay dilepton yield shown in the different papers. i.e. "Ernst" formula is used with* G_M*=3 (fig.2 of [1]), "Wolf " formula is used with* G_M*=5.44 (fig.6 of [3]), "Krivoruchenko" formula with* G_M*=3.02 (fig. 26 of [9]) .*

The increase of the decay width is enhanced in the calculations with Ernst and Wolf's formula (for example the ratio between "Ernst" and "Krivoruchenko" formula is 2.2 for M_Δ=1.5 GeV/c^2 instead of 1.6 for M_Δ=1.232 GeV/c^2. The global effect of this Δ mass effect on the dilepton spectrum depends on the contribution of large Δ masses in the production cross-section.

In this respect, it would be interesting to compare the Δ mass distributions used in the different transport models.

In our PLUTO generator, the pion exchange amplitude induces a four-momentum transfer dependence which modifies the Breit-Wigner Δ mass distribution, suppressing high Δ masses corresponding to higher four-momentum transfers. This is based on Dmitriev's one pion exchange model, which has been checked against pp→pΔ^{++} data. It seems that in the different transport models, such a t-dependence is not taken into account, which might produce a higher contribution of large Δ masses. The quantitative effect of this possible difference has however to be checked. In addition, but may be less important, it would be interesting to check whether the different Breit-Wigner formula used, and the different mass dependent width (some with cut-off function, some without) lead to similar results.

7 Conclusion

The three litteral formulas we have studied have different analytic expressions. We don't know the origin of these differences, but we checked that they provide the same dilepton invariant mass dependence, with normalizations which are however different by up to a factor 4.75. Nevertheless, due to the use of different magnetic form factors, or to some arbitrary renormalisations, the dilepton spectra shown in the papers differ by a factor 1.6 at maximum at the resonance mass M_Δ=1.232 GeV. This normalisation problem should be fixed, using for example the radiative decay width.

"Krivoruchenko"'s formula gives the good relation between radiative Δ decay width and magnetic form factor, which gives confidence that this formula is correct.

In the PLUTO generator, the dilepton mass distribution follows the "Ernst" formula, but the normalisation is independent. Our investigations confirm that, in the case of a point-like N-Δ transition, the Δ Dalitz decay branching ratio should be of the order of 4.2 10^{-5}, therefore very close to the value used actually in PLUTO.

The different formula have also a different Δ mass dependence, the quantitative effect of which depends on the contribution of large Δ masses in the production cross-section. The Δ mass distribution is another possible difference between the calculations which would be worth to investigate, due to the sensitivity of the Dalitz decay dilepton mass spectrum to the Δ mass.

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